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MULTIFACTOR MODEL FOR PRICING CRYPTOCURRENCIES

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ABSTRACT

The main aim of the paper is to contribute to the discussion of cryptocurrency pricing through asset pricing models. The paper develops multifactor models for the cryptocurrency valuation and discusses the elements of these models based on the literature review of scholarly articles, books, and scientific databases. We further developed the models from unit factor to five factors in order to observe whether the development is achieved by adding new factors into the model. Consequently, we pointed out the most influential factor that determines the value of cryptocurrencies.

Keywords: Cryptocurrencies, factor models, asset pricing, momentum strategy, liquidity, crypto market cap.

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INTRODUCTION

This paper focuses on the study of cryptocurrency returns, with the purpose of constructing an asset pricing model with related factors to determine the investment strategy in the cryptocurrency market. We use the Fama Macbeth two pass methodology to analyze the relationship between the factors and cryptocurrency returns. The results indicate that the one-factor model, with the independent variable of MKT Value-Weighted risk factor, does not provide a statistically significant explanation for pricing the cross-section and time series cryptocurrency return. Moreover, additional analysis had been performed, and multiple-factor regression models were employed with explanatory variables of Volatility (VOL), Size (SIZE), and Momentum (MOM) to test whether these variables can statistically explain the crypto-return movements. Nevertheless, neither of the variables in the combination of the factor models were statistically significant to explain the return movements. Lastly, we added an additional factor called Liquidity (LIQ) to consider whether the "missing" variable is the liquidity of cryptocurrencies. We observed that the coefficient for the crypto liquidity is invalid to explain the returns on cryptocurrencies.

In order to construct a liquidity factor, we focused on the returns data for 8 major cryptocurrencies: Bitcoin (BTC), Ethereum (ETH), Litecoin (LTC), Cardano (CRD), Monero (MNR), Dash (DSH), ZCash (ZCH), and Ripple (XRP). We observed monthly trading volume by dividing average monthly trading volume to average market capitalization for each pair of cryptocurrencies. Also, we constructed daily return table from January 1st 2018 to December 31st 2021. Based on the return table, we constructed a high-low portfolio based on the liquidity of the cryptocurrencies by finding 2 most liquid and 2 least liquid asset for each month and found the average liquidity for each of them. Consequently, we regressed crypto returns with their liquidity in order to have an inference regarding the statistical significance of the liquidity factor over crypto returns.

We used Makarov and Schoar (2020) to be informed about the arbitrage opportunities in the crypto market, Griffiin (2020) to gain the detailed operational understanding of the blockchain process, Foley (2019) to obtain knowledge regarding the illegal activity financing via the usage of cryptoassets, Sadka (2006) to determine the role of liquidity risk in post-earning anomalies, Bianchi and Dickerson (2018) to get intuition associated with the trading volumes in major crypto units which are used in determining the statistical significance.

1. DETERMINATION OF THE FACTORS IN CRYPTOCURRENCY PRICING

Unlike traditional currencies, cryptocurrencies are neither issued by a central bank nor backed by governmental agencies, thus the price of these investment tools is not affected by changes in monetary policy tools. As a result, cryptos are generally used as a commodity for the store of value. We consider several following factors that may affect their valuation:

- Sensitivity to the Market Index (MKT-Value Weighted)
- Volatility (VOL)
- Size
- Momentum (MoM)

MKT - Value-Weighted: This factor is akin to a market capitalization-stock index. Constructed as the average return to holding the entire basket of cryptocurrencies in the sample at each time t.

In other words, this "risk factor" is simply the value-weighted return across all cryptocurrencies quoted in USD for each time t. This factor could be considered analogous to the "market risk factor" in stock markets.

VOL: The "Volatility" factor is constructed by first computing the rolling 30-day volatility of each cryptocurrency portfolio in the sample. The cryptocurrencies are then sorted into deciles based on their lagged 30-day volatility. A long position is taken in decile 10 (the high volatility portfolio) and a short position in decile 1 (the low volatility portfolio).

SIZE: The "Size" factor is constructed by sorting cryptocurrencies into deciles based on lagged market capitalization. A long position is taken in decile 1 (the small cap portfolio) and a long position in decile 10 (the large cap portfolio). This factor can be considered analogous to the Fama-French Size factor often abbreviated as SMB (Small-Minus-Big).

MOM: The "Momentum" factor is computed by first computing the rolling 30-day momentum (sum of the log returns for each crypto pair) in the sample. The cryptocurrencies are then sorted into deciles based on lagged 30-day momentum. A long position is taken in decile 10 (the high momentum portfolio) and a short position in decile 1 (the low momentum portfolio).

2. METHODOLOGY

We used Fama-Macbeth two-pass methodology to carry out time series regression to obtain coefficient estimates of alpha and beta for 50 portfolios these inputs are used to carry out cross-sectional regression and obtain gamma coefficients **(Appendix 3)**. We first start from a market factor, and then test other three factors sequentially as a single-factor model. With aim of constructing a multi-factor model with greater explanatory power, we find there are 15 different combinations of four factors. By running those 15 different models, we get 15 groups of R^2 and GRS results, where we conclude all the results **(Appendix 2)**. For each portfolio, we compute t-statistics for alpha and beta coefficients, 50 groups in total for each model. Subsequently, we've decided to add a new related factor which is called liquidity. The methodology we used can be described as follows. Firstly, we downloaded data of daily transaction units, daily prices and market capitalization for 8 coins. The market capitalization for cryptocurrencies can be computed using daily price multiplied by total units mined. Secondly, we sorted 8 coins into four groups, ranked by their monthly average turnover rate (rolling 30-day). Thirdly, we compute factor returns by subtracting daily returns of low liquidity portfolio from daily returns of high liquidity portfolio, where we define portfolio return as arithmetic mean of returns of assets within that portfolio in corresponding trading days. Finally, we put liquidity factor returns (T=1462) in time sequence, insert it into our data file and match with other factors to be used for regression analysis.

3. DATA AND EMPIRICAL RESULTS

3.1. Data description

The data file contains two sets of data, one is 50 portfolios daily log returns and the other one is factor returns with 1462 data from 2018 to 2021. We separate them into two txt files and load them into Matlab, with T = 1462 and N = 50. K depends on the number of factors used in the model, with a range of 1 to 5.

3.2. Preliminary data analysis and summary statistics

We conduct a time-series regression and a t-test to test the portfolio's alphas and β. According to t-statistic of the alpha, there are 20 alphas in the portfolios that are not statistically significant, and there are 30 alphas in portfolios that are statistically significant **(Appendix 3)**. By same test, every β in portfolios is statistically significant. After the time-series regression, we use two-pass Fama-Macbeth to price the cryptocurrencies returns. According to result, we get Υ_0=3.61% and Υ_{mkt}=3.39% **(Appendix 2)**. Here is our explanation why market risk could be negative. On the statistical level, market factor is correlated with other factors. When adding other factors, Υ_{mkt} becomes smaller, still negative but insignificant. So, there is an omitted variable bias. Economically, according to the certainty equivalent theory, risk-averse investors will ask for a positive risk premium for compensating their risk. In contrast, risk lovers can tolerate a negative risk premium to bet for a higher market return. Most of the investors who invested in cryptocurrency are risk lovers, so the price of market risk for those investors could be negative. Then we conduct a GRS-test to test if the portfolios alphas are jointly statistically different to zero. The result showed that GRS= 4.6133 **(Appendix 1)**, which rejects the null hypothesis, the portfolios alphas are not jointly equal to zero. However, CAPM-based theory required alphas equal to zero, the result of GRS-test contradicted CAPM-based theory. Therefore, it is not wise to apply CAPM-based theory to price cryptocurrencies by MKT factor model since there are pricing errors. We also use t-test to test significance of Υ_0 and Υ_{mkt}. According to the result, Υ_{mkt} is statistically significant while Υ_0 is not. However, we get the time-series R^2 equals 0.2386 and cross-sectional R^2 equals 0.3401 **(Appendix 1)**, both are much lower than 1, which means that MKT factor model is not enough to explain cryptocurrency returns well, we still need other factors to explain it. Overall, it's shown that MKT factor model is not suitable for pricing the cryptocurrency returns due to its high GRS-statistics and low R^2.

We compared four models with only single factor, among all single factor models, the size factor provides highest explanatory power to other three factors, as indicated by its t-statistics significance and compared low GRS-statistics of 3.0683 **(Appendix 1)**. The results have shown that using size factor alone may outperform several multi- factor models, thus leaving other factors redundant. Apart from the statistics result, Liu, Tsyvinski and Wu（2022) also states that small size cryptocurrencies have lower price and higher illiquidity relative to larger coins, where people tend to believe a lottery that small size coin will give higher return in the future. In addition, risk premium for the cross-sectional size factor model is 0.0114, which shows that an increasing correlation between coin size and portfolio return will give a higher return. This can be explained by the theory stated by Liu, Tsyvinski and Wu (2022) that the cryptocurrency premium is more pronounced among coins that have high arbitrage cost. As for the risk premium of momentum and volatility, the momentum gives a positive risk premium in the single factor model, with 0.0209, this is in line with the investor overreaction channel e.g Daniel, Hirshleifer, and Subrahmanyam (1998), Sockin and Xiong (2018)). Volatility gives a negative risk premium with -0.0358 due to omitted variable bias **(Appendix 2)**. There are true relevant factors excluded from the model, and due to the fact that market and size factor premiums are positively correlated with volatility factor return **(Appendix 8 & 9)**, the estimated beta of volatility has been overestimated in the single factor model.

3.3. *Description and discussion of result*

Volatility factor does not seem to be significant, and economic intuitions which could explain this is that since cryptocurrency is one of the most volatile assets in the financial market, simply adding volatility does not guarantee a higher risk premium compensated to the investor. In the CAPM model, expected stock return is proportional to systematic volatility, while taking idiosyncratic risk cannot be rewarded. If volatility is insignificant, then the difference between cryptocurrency return may represent an idiosyncratic component. Given the low R^2 of 0.0873 **(Appendix 1)**, clearly the volatility factor of cryptocurrency cannot act as a good proxy of risk and return. However, there is a large proportion of cryptocurrency return that can be explained by momentum factor in the cross-sectional regression, with $R^2 = 0.7371$, only 26.29% variation is left unexplained. This phenomenon is in line with the nature of the cryptocurrency 7x24 trading period and the use of algorithm trading techniques. As the price of cryptocurrency is non-stationary, patterns of trend can be easily observed. Investors long the cryptocurrency with signals of upward movement, these signals are captured by computers which attract buy orders to push-up cryptocurrency prices. This is consistent with the statement from Liu, Tsyvinski and Wu (2022) that momentum effect arise from investors' delayed reaction or overreaction to information.

To extend the single-factor model to a multi-factor one, we start from a size factor model to insert other factors and form two, three or four-factor models. By adding volatility factor, it has seen a slight decrease in the GRS statistics. Nevertheless, other combinations of two-factor models fail to achieve lower GRS statistics. Among three- factor models, the best one is vol+size+mom with no doubt, and it certainly beats the performance of the four-factor model by looking at the GRS statistics. Its cross- sectional R^2 of 0.7729 is fairly close to 0.7833 in the four-factor model.

The factor return of our new liquidity factor is, on average, positive. This means cryptocurrencies with higher liquidity outperformed those with lower liquidity in our sample period. Our preliminary results show that liquidity factor alone cannot explain 50 portfolio returns **(Appendix 7)**, because it has negative gamma and low R. Clearly, we need to add more explanatory variables to improve the power of the model. In our multi-factor model, the gamma of liquidity factor becomes positive **(Appendix 11)**, although it's insignificant in some cases. Adding the liquidity factor does not consistently reduce GRS statistics for all possible combinations **(Appendix 10)**, the problem of multicollinearity sometimes makes it redundant. It's reasonable that the liquidity factor has positive correlations with volatility and momentum factor, as indicated by our VCV matrix and correlation matrix. This can be illustrated when a cryptocurrency experiences a bull market, it attracts more attention from investors, higher trading volume is associated with greater price fluctuations, thus both volatility and liquidity rise. Meanwhile, the momentum factor could also explain some of the returns under this situation. Another reason why liquidity factor seems to be relevant but not always helpful in the regression is that we have some limitations in our sorting methodology. Our sample size for coins is relatively small, ideally, we may collect data for all cryptocurrencies currently trading in market, though it's unrealistic at this moment. When constructing this liquidity factor, the volume data for only 8 coins have been taken into the sorting process, and there are only two assets in one group, so it's insufficient to represent the whole market. If we include more assets in one group and increase number of groups we constructed, the factor return would be more significant and less volatile. Since the factor return itself is too volatile, it potentially leads to huge error terms in our time series regression, thus making gamma coefficient estimate to be insignificant (A possible Type-II error where we fail to reject a false null hypothesis). Furthermore, the monthly rolling period may be too long for timely adjustment of the portfolio. As turnover rate of cryptocurrencies is varying quickly, this means that some changes in ranking of liquidity within every month may be omitted in our computation.

CONCLUSION AND FUTURE CHALLENGES

We conclude that cryptocurrency returns have low or little exposures to market index, volatility, size, momentum, and liquidity factors, both individually and together. We observed that our multi-factor model overall is invalid in explaining crypto-returns, and the inclusion of liquidity factor does not generate a statistically significant explanation for cryptocurrencies portfolio return variation. From the perspective of R^2, the five-factor model wins but this situation will change if we use adjusted R^2 instead. In our multi-factor model, the combination of size and volatility factor yields the lowest GRS-stat. Size factor is significant at all conventional significance levels, no matter on its own or combining with other factors. By comparing the statistics in Appendix 1 and 10, there is no significant improvement given by liquidity factor. Consequently, for future research purposes, would need to explore other new factors to build our asset pricing model before making investment decisions.

The paper is far from being perfect and has some limitations. One of the major limitations that the paper has is that in determining the statistical significance of the liquidity factor, the sample size was chosen to be 8 cryptocurrencies due to the data limitation, which is clearly small. On the other hand, the chosen cryptocurrencies themselves may reflect sample bias and result in a fairly erroneous interpretation. We believe these limitations can be the motivation for the new research and some of them are already topics of the ongoing debate in construction of the asset pricing model for the cryptocurrencies.

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APPENDIX

APPENDIX 1

GRS JOINT SIGNIFICANT TEST RESULTS

	TS R^2	CS R^2	GRS
MKT	0.3401	0.2386	4.6133
VOL	0.0105	0.0837	3.7055
SIZE	0.027	0.5759	3.0683
MOM	0.0027	0.7371	4.0736
VOL+MKT	0.0105	0.2779	4.5450
SIZE+MOM	0.0289	0.7727	3.3463
SIZE+MKT	0.3682	0.5870	3.8874
MOM+MKT	0.3422	0.7648	4.9630
VOL+SIZE	0.0354	0.5761	2.9607
VOL+MOM	0.0125	0.7378	4.3152
VOL+MKT+MOM	0.3472	0.7650	4.8932
VOL+SIZE+MOM	0.0371	0.7729	3.2334
VOL+SIZE+MKT	0.3723	0.5871	3.7653
SIZE+MOM+MKT	0.3703	0.7831	4.2018
MOM+MKT+SIZE+VOL	0.3743	0.7833	4.0742

APPENDIX 2

TWO-PASS FAMA MACBETH REGRESSION RESULTS

	Gamma coefficients							T-statistics for						
	γ0	γ1	γ2	γ3	γ4	γ5	γ6	γ0	γ1	γ2	γ3	γ4	γ5	γ6
MKT	0.0361	-0.0339						8.16	-7.97					
VOL	0.0038	-0.0359						2.75	-5.70					
SIZE	-0.0038	0.0114						-2.81	10.5					
MOM	0	0.0209						-0.05	11.2					
VOL +MKT	0.0359	-0.0320	-0.027					8.12	-7.65	-4.56				
SIZE +MOM	-0.0014	0.0084	0.017					-1.06	7.63	7.18				
SIZE +MKT	0.0057	0.0106	-0.0087					1.26	9.28	-2.06				
MOM +MKT	0.0133	0.0192	-0.0124					3.09	10.1	-2.97				
VOL +SIZE	- 0.0036	0	0.011					2.51	0.10	9.82				
VOL +MOM	0	-0.004	0.02					0.20	-0.69	10.9				
VOL +MKT +MOM	0.01	-0.004	0.0191	-0.012				3.07	- 0.56	9.85	- 2.98			
VOL +SIZE +M OM	- 0.002	0.002	0.0165	0.008				- 1.19	0.37	7.14	7.61			
VOL +SIZE M KT	0.006	0	0.0104	-0.081				1.29	- 0.05	8.74	- 2.07			
SIZE +MOM+M KT	0.008	0.016	0.0075	-0.008				1.72	7.11	6.58	- 1.94			
MOM+M KT+ SIZE +VO L	0.008	0.0013	0.0163	0.0707	- 0.0082			1.66	0.22	7.07	6.59	-1.92		

APPENDIX 3
TIME SERIES ALPHA AND BETA FOR TIME-SERIES OF MARKET FACTOR MODEL IN SINGLE FACTOR MODEL

	MKT's alpha	t(alpha)	MKT's beta	t(beta)	R^2
1	0.0143	6.34	0.9311	17.31	0.1708
2	0.0132	5.14	0.9124	14.95	0.1333
3	0.0117	4.99	1.0071	18.05	0.1830
4	0.0055	2.26	1.0162	17.41	0.1724
5	0.0076	3.01	0.9842	16.32	0.1549
6	0.0013	0.57	0.9413	17.91	0.1806
7	0.006	2.73	1.0412	19.80	0.2122
8	0.0005	0.25	1.1512	24.45	0.2911
9	0.0000	0.02	1.0084	18.90	0.1971
10	-0.001	-0.45	0.9663	18.56	0.1914
18	-0.0037	-1.63	1.0311	19.15	0.2013
20	-0.0039	-1.94	1.0255	21.39	0.2391
26	-0.0037	-1.92	1.0589	22.91	0.2649
27	-0.0041	-2.16	1.0614	23.73	0.2788
28	-0.0017	-0.82	1.105	22.48	0.2576
29	-0.0045	-2.46	1.1185	25.71	0.3121
31	-0.0024	-1.15	1.1592	23.10	0.2682
33	-0.0036	-1.85	1.0332	22.57	0.2592
34	-0.0067	-3.66	1.0843	24.83	0.2974
35	-0.0039	-2.03	1.0885	23.62	0.2770
36	-0.0044	-2.32	1.0268	22.57	0.2591
39	-0.0042	-2.3	1.1222	26.08	0.3182
41	-0.0008	-0.44	1.0505	25.14	0.3026
42	-0.0041	-2.25	1.0028	22.89	0.2646
43	-0.0039	-2.08	1.0925	24.77	0.2963
44	-0.0057	-3.32	1.0762	26.28	0.3215
49	-0.0059	-3.03	1.0167	21.98	0.2491
51	-0.0023	-1.21	1.1002	24.76	0.2962
56	-0.004	-2.33	1.0268	24.88	0.2982
60	-0.0028	-1.65	1.0773	26.72	0.3289
61	-0.0044	-2.28	1.1506	25.24	0.3043
63	-0.0029	-1.72	1.1308	28.36	0.3557
72	-0.0027	-1.59	1.1607	29.14	0.3681
73	-0.0028	-1.65	1.1479	28.09	0.3513
74	-0.0012	-0.76	1.0388	27.17	0.3362
81	-0.0058	-3.99	0.9781	28.12	0.3518
82	-0.0023	-1.53	1.1142	31.18	0.4002
84	-0.0038	-2.47	1.0669	29.45	0.3730
87	-0.0037	-2.73	1.077	33.76	0.4388
88	-0.0024	-1.64	1.0917	31.94	0.4117
91	-0.0031	-2.63	1.1451	41.21	0.5380
92	-0.0034	-2.61	1.1466	36.94	0.4835
93	-0.0027	-2.47	1.1252	43.29	0.5624
94	-0.0026	-2.04	1.0917	36.62	0.4791
95	-0.0013	-1.05	1.047	35.67	0.4660
96	-0.0033	-3.14	1.0148	39.97	0.5228
97	-0.0023	-2.2	1.0665	43.21	0.5615
98	-0.0021	-1.99	1.1515	46.74	0.5997
99	-0.0035	-3.72	1.1975	52.87	0.6572
100	-0.0007	-3.49	0.9883	204.20	0.9662

APPENDIX 4

THE ALPHA AND BETA VALUE FOR THE TIME SERIES OF THE VOLATILITY IN SINGLE FACTOR MODEL

	Vol's alpha	t(alpha)	Vol's beta	t(beta)	R^2
1	0.0159	6.4373	0.0821	2.2506	0.0041
2	0.0148	5.3961	0.1015	2.5095	0.0050
3	0.0134	5.1596	0.0683	1.7918	0.0029
4	0.0072	2.6910	0.0706	1.7802	0.0028
5	0.0093	3.3947	0.0860	2.1233	0.0038
6	0.0029	1.1919	0.0837	2.3360	0.0044
7	0.0077	3.0779	0.0420	1.1482	0.0016
8	0.0024	1.0345	0.0808	2.3419	0.0044
9	0.0016	0.6316	0.0248	0.6741	0.0010
10	0.0007	0.2993	0.0942	2.6366	0.0054
18	-0.0019	-0.7516	0.0870	2.3406	0.0044
20	-0.0020	-0.8697	0.1253	3.7033	0.0100
26	-0.0018	-0.8170	0.1045	3.1453	0.0074
27	-0.0021	-0.9620	0.1175	3.6239	0.0096
28	0.0003	0.1266	0.1148	3.2667	0.0079
29	-0.0026	-1.1672	0.0930	2.8756	0.0063
31	-0.0003	-0.1163	0.1350	3.7396	0.0102
33	-0.0017	-0.7519	0.1166	3.5624	0.0093
34	-0.0048	-2.2014	0.1011	3.1477	0.0074
35	-0.0020	-0.8671	0.1120	3.3545	0.0083
36	-0.0025	-1.1146	0.1428	4.3963	0.0137
39	-0.0023	-1.0348	0.0837	2.6033	0.0053
41	0.0010	0.4847	0.0821	2.6613	0.0055
42	-0.0024	-1.1366	0.0781	2.4763	0.0049
43	-0.0019	-0.8675	0.1102	3.4029	0.0085
44	-0.0040	-1.8969	0.0592	1.9290	0.0032
49	-0.0041	-1.8192	0.1027	3.1217	0.0073
51	-0.0002	-0.1075	0.1244	3.8182	0.0106
56	-0.0022	-1.0678	0.1064	3.5076	0.0090

APPENDIX 5

ALPHA AND BETA VALUE FOR TIME SERIES OF MOMENTUM IN SINGLE FACTOR MODEL

	MOM's alpha	t(alpha)	MOM's beta	t(beta)	R^2
1	0.0157	6.3393	0.2766	2.6003	0.0053
2	0.0145	5.3205	0.5927	5.0529	0.0179
3	0.0131	5.1008	0.4080	3.6792	0.0099
4	0.0070	2.6173	0.3783	3.2791	0.0080
5	0.0091	3.3295	0.6245	5.3281	0.0197
6	0.0026	1.0723	0.0875	0.8348	0.0012
7	0.0075	3.0423	0.3672	3.4478	0.0088
8	0.0021	0.9141	0.0641	0.6358	0.0010
9	0.0015	0.5997	0.0912	0.8489	0.0012
10	0.0004	0.1615	-0.0222	-0.2123	0.0007
18	-0.0022	-0.8785	-0.1946	-1.7917	0.0029
20	-0.0024	-1.0633	-0.1450	-1.4624	0.0021
26	-0.0022	-0.9769	0.0644	0.6623	0.0010
27	-0.0025	-1.1556	-0.2344	-2.4712	0.0048
28	-0.0001	-0.0496	-0.2527	-2.4600	0.0048
29	-0.0029	-1.3168	-0.0681	-0.7194	0.0010
31	-0.0008	-0.3086	0.0453	0.4276	0.0008
33	-0.0021	-0.9356	-0.0455	-0.4737	0.0008
34	-0.0052	-2.3646	-0.1130	-1.2023	0.0017
35	-0.0024	-1.0382	0.0275	0.2814	0.0007
36	-0.0030	-1.3383	-0.0725	-0.7598	0.0011
39	-0.0026	-1.1713	-0.0746	-0.7929	0.0011
41	0.0007	0.3460	0.0005	0.0056	0.0007
42	-0.0027	-1.2680	-0.1080	-1.1719	0.0016
43	-0.0023	-1.0406	0.0386	0.4072	0.0008
44	-0.0042	-2.0065	-0.1976	-2.2053	0.0040
49	-0.0044	-1.9905	-0.2741	-2.8525	0.0062
51	-0.0007	-0.3071	-0.0777	-0.8132	0.0011
56	-0.0026	-1.2528	-0.1627	-1.8316	0.0030
60	-0.0012	-0.6034	-0.0055	-0.0617	0.0007
61	-0.0027	-1.1880	-0.0819	-0.8305	0.0012
63	-0.0013	-0.6035	-0.0017	-0.0193	0.0007
72	-0.0010	-0.4820	-0.1384	-1.5315	0.0023
73	-0.0012	-0.5617	-0.0144	-0.1576	0.0007
74	0.0003	0.1301	-0.0825	-0.9745	0.0013
81	-0.0044	-2.4466	-0.0309	-0.3968	0.0008
82	-0.0007	-0.3602	-0.0188	-0.2255	0.0007
84	-0.0022	-1.1648	-0.1260	-1.5276	0.0023
87	-0.0021	-1.1836	-0.0586	-0.7626	0.0011
88	-0.0008	-0.4280	-0.0851	-1.0589	0.0015
91	-0.0014	-0.8320	0.0038	0.0512	0.0007
92	-0.0018	-0.9719	-0.0208	-0.2670	0.0007
93	-0.0011	-0.6606	-0.0318	-0.4479	0.0008
94	-0.0010	-0.5707	0.0046	0.0623	0.0007
95	0.0002	0.1257	0.0297	0.4093	0.0008
96	-0.0019	-1.2314	-0.0310	-0.4669	0.0008
97	-0.0008	-0.4866	-0.0687	-1.0225	0.0014
98	-0.0004	-0.2503	0.0345	0.4903	0.0008
99	-0.0018	-1.1260	-0.0239	-0.3419	0.0008
100	0.0007	0.6375	0.0038	0.0799	0.0007

APPENDIX 6

REGRESSION WITH SIZE FACTOR – RESULTS

	Size's alpha	t(alpha)	Size's beta	t(beta)	R^2
1	0.0105	4.4188	0.8863	12.8052	0.1016
2	0.0088	3.3268	0.9802	12.7542	0.1009
3	0.0084	3.3152	0.8253	11.2796	0.0808
4	0.0017	0.6705	0.9080	12.0024	0.0904
5	0.0028	1.0741	1.0764	14.1661	0.1214
6	-0.0021	-0.8698	0.8061	11.7533	0.0871
7	0.0022	0.9262	0.9171	13.2593	0.1081
8	-0.0022	-0.9564	0.7476	11.2821	0.0808
9	-0.0038	-1.5956	0.9191	13.2148	0.1074
10	-0.0053	-2.3121	0.9871	14.7916	0.1309
18	-0.0040	-1.5798	0.3147	4.2529	0.0129
20	-0.0039	-1.6567	0.2450	3.6235	0.0096
26	-0.0043	-1.8733	0.3534	5.3561	0.0199
27	-0.0039	-1.7524	0.2380	3.6729	0.0098
28	-0.0015	-0.6259	0.2439	3.4742	0.0089
29	-0.0044	-1.9624	0.2533	3.9282	0.0111
31	-0.0023	-0.9331	0.2691	3.7305	0.0101
33	-0.0038	-1.7130	0.3050	4.6761	0.0154
34	-0.0064	-2.8900	0.2119	3.3034	0.0081
35	-0.0038	-1.6463	0.2451	3.6773	0.0099
36	-0.0044	-1.9659	0.2486	3.8249	0.0106
39	-0.0047	-2.1548	0.3748	5.8892	0.0239
41	-0.0003	-0.1329	0.1744	2.8287	0.0061
42	-0.0036	-1.6657	0.1563	2.4815	0.0049
43	-0.0029	-1.3064	0.1077	1.6594	0.0026
44	-0.0050	-2.3692	0.1448	2.3614	0.0045
49	-0.0054	-2.3748	0.1651	2.5083	0.0050
51	-0.0020	-0.8728	0.2218	3.4015	0.0085
56	-0.0039	-1.8831	0.2338	3.8598	0.0108
60	-0.0026	-1.2497	0.2349	3.8836	0.0109
61	-0.0044	-1.9259	0.2993	4.4624	0.0141
63	-0.0022	-1.0328	0.1592	2.6000	0.0053
72	-0.0021	-0.9787	0.1862	3.0161	0.0069
73	-0.0028	-1.2982	0.2749	4.4149	0.0138
74	-0.0011	-0.5779	0.2436	4.2251	0.0128
81	-0.0055	-3.0310	0.1929	3.6299	0.0096
82	-0.0016	-0.8008	0.1509	2.6531	0.0055
84	-0.0028	-1.4406	0.0996	1.7632	0.0028
87	-0.0031	-1.6946	0.1643	3.1336	0.0074
88	-0.0020	-1.0646	0.2100	3.8325	0.0106
91	-0.0023	-1.3492	0.1578	3.1336	0.0074
92	-0.0026	-1.4215	0.1464	2.7494	0.0058
93	-0.0025	-1.4927	0.2401	4.9861	0.0174
94	-0.0018	-1.0226	0.1390	2.7303	0.0058
95	-0.0003	-0.1610	0.0839	1.6912	0.0026
96	-0.0025	-1.6167	0.1085	2.3926	0.0046
97	-0.0009	-0.5661	0.0245	0.5312	0.0009
98	-0.0006	-0.3908	0.0409	0.8499	0.0012
99	-0.0019	-1.1420	0.0094	0.1960	0.0007
100	0.0011	0.9560	-0.0633	-1.9473	0.0033

APPENDIX 7

THE ALPHA AND BETA FOR LIQUIDITY

	LIQ's alpha	t(alpha)	LIQ's beta	t(beta)	R^2
1	0.0156	6.1700	0.0048	0.0672	0.0007
2	0.0135	4.8120	0.1312	1.6717	0.0026
3	0.0133	5.0401	-0.0288	-0.3890	0.0008
4	0.0067	2.4501	0.0352	0.4585	0.0008
5	0.0087	3.1106	0.0397	0.5059	0.0009
6	0.0023	0.9266	0.0409	0.5887	0.0009
7	0.0077	3.0473	-0.0312	-0.4403	0.0008
8	0.0023	0.9426	-0.0162	-0.2424	0.0007
9	0.0010	0.3942	0.0663	0.9298	0.0013
10	-0.0004	-0.1666	0.1100	1.5872	0.0024
18	-0.0028	-1.0763	0.0780	1.0812	0.0015
20	-0.0026	-1.0892	0.0170	0.2586	0.0007
26	-0.0026	-1.1333	0.0553	0.8557	0.0012
27	-0.0026	-1.1308	0.0026	0.0411	0.0007
28	-0.0004	-0.1681	0.0420	0.6142	0.0009
29	-0.0026	-1.1642	-0.0376	-0.5987	0.0009
31	-0.0004	-0.1758	-0.0436	-0.6210	0.0009
33	-0.0028	-1.2254	0.0963	1.5130	0.0022
34	-0.0060	-2.6826	0.1125	1.8043	0.0029
35	-0.0033	-1.4077	0.1232	1.9005	0.0032
36	-0.0029	-1.2647	-0.0133	-0.2099	0.0007
39	-0.0042	-1.8861	0.2234	3.5943	0.0094
41	0.0010	0.4490	-0.0323	-0.5382	0.0009
42	-0.0039	-1.7909	0.1639	2.6845	0.0056
43	-0.0034	-1.5286	0.1555	2.4733	0.0049
44	-0.0045	-2.1110	0.0452	0.7593	0.0011
49	-0.0051	-2.2357	0.0931	1.4573	0.0021
51	-0.0016	-0.7154	0.1288	2.0322	0.0035
56	-0.0022	-1.0647	-0.0448	-0.7589	0.0011
60	-0.0018	-0.8725	0.0810	1.3745	0.0020
61	-0.0023	-0.9929	-0.0535	-0.8180	0.0011
63	-0.0027	-1.2667	0.1949	3.2871	0.0080
72	-0.0014	-0.6496	0.0533	0.8882	0.0012
73	-0.0015	-0.6962	0.0435	0.7161	0.0010
74	-0.0004	-0.1856	0.0864	1.5389	0.0023
81	-0.0048	-2.6017	0.0523	1.0116	0.0014
82	-0.0022	-1.0958	0.1992	3.6206	0.0096
84	-0.0031	-1.6007	0.1237	2.2609	0.0042
87	-0.0020	-1.0922	-0.0160	-0.3127	0.0008
88	-0.0018	-0.9373	0.1352	2.5381	0.0051
91	-0.0026	-1.4702	0.1555	3.1852	0.0076
92	-0.0025	-1.3430	0.0986	1.9088	0.0032
93	-0.0029	-1.7264	0.2445	5.2437	0.0192
94	-0.0032	-1.8447	0.3046	6.2371	0.0266
95	-0.0013	-0.7429	0.2020	4.2225	0.0127
96	-0.0030	-1.8956	0.1470	3.3515	0.0083
97	-0.0012	-0.7355	0.0570	1.2767	0.0018
98	-0.0018	-1.0925	0.1911	4.1215	0.0122
99	-0.0031	-1.8863	0.1754	3.8039	0.0105
100	0.0011	0.9939	-0.0568	-1.8023	0.0029

APPENDIX 8

VCV MATRIX FOR THE FACTORS

VCV	MKT	VOL	SIZE	MOM	LIQ
MKT	**0.1760%**	0.0151%	-0.0019%	0.0005%	0.0008%
VOL	0.0151%	**0.4599%**	0.0208%	-0.0024%	0.0129%
SIZE	-0.0019%	0.0208%	**0.1152%**	0.0142%	0.0079%
MOM	0.0005%	-0.0024%	0.0142%	**0.0540%**	0.0031%
LIQ	0.0008%	0.0129%	0.0079%	0.0031%	**0.1226%**

APPENDIX 9

CORR. MATRIX FOR THE FACTORS

Cor	MKT	VOL	SIZE	MOM	LIQ
MKT	**1.00**	0.05	-0.01	0.01	0.01
VOL	0.05	**1.00**	0.09	-0.02	0.05
SIZE	-0.01	0.09	**1.00**	0.18	0.07
MOM	0.01	-0.02	0.18	**1.00**	0.04
LIQ	0.01	0.05	0.07	0.04	**1.00**

APPENDIX 10

THE TIME SERIES AND CROSS-SECTIONAL R^2 VALUE FOR EACH MODEL AND THEIR GRS- RESULT

	TS R^2	CS R^2	GRS
LIQ	0.0040	0.0397	4.5346
LIQ + MOM	0.0060	0.7511	4.9166
LIQ + SIZE	0.0300	0.5760	3.9930
LIQ + VOL	0.0134	0.1131	4.4391
LIQ + MKT	0.2477	0.2477	5.1840
LIQ+ SIZE+MOM	0.0319	0.7771	4.3322
LIQ+SIZE+MKT	0.3709	0.5871	4.6075
LIQ+MKT+MOM	0.3452	0.7725	5.5980
LIQ+SIZE+VOL	0.0380	0.5761	3.8428
LIQ+VOL+MKT	0.3480	0.2850	5.0762
LIQ+VOL+MOM	0.0154	0.7514	4.8190
LIQ+MKT+SIZE+MO M	0.3731	0.7863	4.9780
LIQ+MKT+VOL+MO M	0.3501	0.7725	5.4883
LIQ+SIZE+VOL+MK T	0.3749	0.5873	4.4435
LIQ+SIZE+VOL+MO M	0.0398	0.7773	4.1751
LIQ + SIZE +MOM +MKT+VOL	0.3769	0.7866	4.8069

APPENDIX 11

THE GAMMA COEFFICIENT OF EACH FACTOR IN EACH MODEL AND THEIR T-STAT

	Gamma coefficients						T-statistics for gamma					
	$\gamma 0$	$\gamma 1$	$\gamma 2$	$\gamma 3$	$\gamma 4$	$\gamma 5$	$\gamma 0$	$\gamma 1$	$\gamma 2$	$\gamma 3$	$\gamma 4$	$\gamma 5$
LIQ	0.009	- 0.01					0.673	- 3.600				
LIQ + MOM	0.004	-0.005	0.02				0.326	-1.750	11.0			
LIQ + SIZE	0.003	0.009	0.11				-2.754	0.321	9.83			
LIQ + VOL	0.004	-0.01	-0.03				3.086	-3.445	-5.53			
LIQ + MKT	0.035	-0.005	-0.03				7.723	-0.1753	-7.36			
LIQ+ SIZE +MOM	-0.001	-0.003	0.0078	0.016			-0.722	-0.832	6.62	7.251		
LIQ+ SIZE +MKT	0.005	0.0014	0.01	-0.008			1.266	0.461	9.11	-2.033		
LIQ+ MKT +MOM	0.012	-0.003	-0.0121	0.019			2.765	-0.121	-2.55	10.06		
LIQ+ SIZE +VOL	- 0.003	0.0009	0.0113	0.000			- 2.494	0.307	9.41	0.109		
LIQ+ VOL +MKT	0.034	- 0.0054	- 0.0275	- 0.030			7.706	- 1.773	- 4.52	- 7.092		
LIQ+ VOL +MOM	0.000	- 0.005	- 0.0038	0.020			0.469	- 1.752	- 0.61	10.74		
LIQ+ MKT +SIZE+M OM	0.007	- 0.00	- 0.00	0.007	0.016		1.689	- 0.677	- 1.79	6.007	7.162	
LIQ+ MKT +VO L +M OM	0.012	- 0.00	- 0.01	- 0.003	0.019		2.740	- 1.214	2.55	- 0.05	9.822	
LIQ+ SIZE +VOL +M OM	-0.001	- 0.00	0.000	0.001	0.016		- 0.880	- 0.815	6.77	0.308	7.208	
LIQ+ SIZE +VOL+ M KT	0.005	0.0013	0.10	- 0.002	- 0.008		1.295	0.447	8.70	-0.08	- 2.041	
LIQ + SIZE +MOM +MKT+VOL	0.007	- 0.002	0.00	0.016	- 0.007	0.0010	1.623	- 0.661	6.14	7.120	- 1.774	0.1733